Lens Distortion

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1 Pinhole camera model

Let $\boldsymbol{X}_c = [X_c, Y_c, Z_c]^\top$ denotes a point in the camera reference frame and $\boldsymbol{x} = [x, y]^\top$ denotes its projection onto the image plane in the camera coordinate. The homogeneous coordinate of a point is described by its tilde as $\tilde{\boldsymbol{x}} = [\boldsymbol{x}^\top, 1]^\top$.

Intrinsic parameters. The intrinsic parameters transform a point in 3D coordinate into the pixel coordinate. The point $\boldsymbol{X}_c$ is projected onto the canonical image plane as

$$\boldsymbol{x}_n = \begin{bmatrix} x_n \\ y_n \end{bmatrix} = \frac{1}{Z_c} \begin{bmatrix} X_c \\ Y_c \end{bmatrix}. \tag{1}$$

Following a polynomial lens distortion model Faugeras and Toscani [1986]; Weng et al. [1992], the lens distorted point $\boldsymbol{x}_d = [x_d, y_d]^\top$ is described as

$$\begin{aligned}
\boldsymbol{x}_d &= D(\boldsymbol{x}_n, \boldsymbol{\delta}) && (2)\\
&= \boldsymbol{x}_n + \boldsymbol{d} && (3)\\
&= \boldsymbol{x}_n + \boldsymbol{d}_{\mathrm{rad}} + \boldsymbol{d}_{\tan}, && (4)\\
\boldsymbol{d}_{\mathrm{rad}} &= \begin{bmatrix} (\delta_1 r^2 + \delta_2 r^4 + \delta_5 r^6) x_n \\ (\delta_1 r^2 + \delta_2 r^4 + \delta_5 r^6) y_n \end{bmatrix}, && (5)\\
\boldsymbol{d}_{\tan} &= \begin{bmatrix} 2\delta_3 x_n y_n + \delta_4 (3x_n^2 + y_n^2) \\ 2\delta_4 x_n y_n + \delta_3 (x_n^2 + 3y_n^2) \end{bmatrix}, && (6)\\
r &= \sqrt{x_n^2 + y_n^2}, && (7)
\end{aligned}$$

where D denotes a lens distortion function that distorts $\boldsymbol{x}_n$ given lens distortion parameter $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_5]^\top$ and $\boldsymbol{d}_{\mathrm{rad}}$ and $\boldsymbol{d}_{\tan}$ denote the radial distortion and tangential distortion vector respectively. The final pixel coordinate $\boldsymbol{x}$ is described using calibration matrix $\boldsymbol{A} \in \mathbb{R}^{3\times 3}$ as

$$\boldsymbol{A} = \begin{bmatrix} f_x & \theta & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$\tilde{\boldsymbol{x}} = \boldsymbol{A} \tilde{\boldsymbol{x}}_d, \tag{9}$$

where $[f_x, f_y]^\top$ denotes the focal length along x and y axes respectively, θ the skew parameter, and $[o_x, o_y]^\top$ the principle point.

Extrinsic parameters. The extrinsic parameters transform the 3D world coordinate to the 3D camera reference coordinate. A point $\boldsymbol{X}_c$ in the world coordinate is transformed to one in the camera reference coordinate by extrinsic parameters as

$$\boldsymbol{X}_c = [\boldsymbol{R}\ \ \boldsymbol{t}]\boldsymbol{X} \tag{10}$$

$$= \boldsymbol{R}\boldsymbol{X} + \boldsymbol{t} \tag{11}$$

$$\rightarrow \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_1 \\ R_{21} & R_{22} & R_{23} & t_2 \\ R_{31} & R_{32} & R_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{12}$$

$$= \begin{bmatrix} R_{11}X + R_{12}Y + R_{13}Z + t_1 \\ R_{21}X + R_{22}Y + R_{23}Z + t_2 \\ R_{31}X + R_{32}Y + R_{33}Z + t_3 \end{bmatrix} \tag{13}$$

$$\lambda\tilde{\boldsymbol{x}}_n = [\boldsymbol{R}\ \ \boldsymbol{t}]\tilde{\boldsymbol{X}} \tag{14}$$

where $\boldsymbol{R} \in \mathbb{R}^{3\times3}$ denotes the rotation matrix and $\boldsymbol{t} \in \mathbb{R}^3$ denotes the translation vector.

Considering the all above components (Eqs. (1), (3), (9), and (10)), the 2D pixel position of a given 3D point is obtained with a 3D point projection function $\mathrm{Proj}(\cdot)$ as

$$\tilde{\boldsymbol{x}}_p = \mathrm{Proj}(\tilde{\boldsymbol{X}}, \boldsymbol{A}, \boldsymbol{\delta}, \boldsymbol{R}, \boldsymbol{t}) \tag{15}$$

$$= \boldsymbol{A}\left[D([\boldsymbol{R}\ \ \boldsymbol{t}]\tilde{\boldsymbol{X}}, \boldsymbol{\delta})\right] \tag{16}$$

$$= \boldsymbol{A}\left[[\boldsymbol{R}\ \ \boldsymbol{t}]\tilde{\boldsymbol{X}} + \tilde{\boldsymbol{d}}\right] \tag{17}$$

$$= \boldsymbol{A}\left[\tilde{\boldsymbol{x}}_n + \tilde{\boldsymbol{d}}\right] \tag{18}$$

2 Projection matrix and lens distortion

In Salvi's survey paper on camera calibration Salvi et al. [2002] that compares several calibration method Faugeras and Toscani [1986]; Hall et al. [1982]; Tsai [1987]; Weng et al. [1992], two important observations are mentioned. One observation is that the calibration method considering lens distortion Tsai [1987]; Weng et al. [1992] provides more accurate result than ones do not consider lens distortion Tsai [1987]; Weng et al. [1992]. The other observation is that a method solving projection matrix Hall et al. [1982] performs better than one solving intrinsic and extrinsic parameters Faugeras and Toscani [1986] because it computes the transformation matrix without any constraint. From these two observations, we are wondering if there is a method that solves projection matrix considering lens distortion.

Here, we consider rewriting Eq. (18) with a projection matrix $\boldsymbol{P} \equiv \boldsymbol{A}[\boldsymbol{R}\ \ \boldsymbol{t}]$. Expanding Eq. (18), we obtain

Algorithm 1 Single cameras calibration algorithm.

Input: Corresponding points $\{\boldsymbol{X}_j \leftrightarrow \boldsymbol{x}_{i,j}\}$.

Step1: Solve intrinsic and extrinsic parameters

```
for i = 1 to N do
    Compute a homography H_i given {X_j ↔ x_{i,j}}.
end for
Compute intrinsic parameters A.
for i = 1 to N do
    Compute extrinsic parameters (R_i, t_i).
end for
Estimate lens distortion parameters δ and refine A by minimizing
```

$$\sum_i \sum_j \tilde{\boldsymbol{x}}_{p,i,j} - \mathrm{Proj}(\tilde{\boldsymbol{X}}_j, \boldsymbol{A}, \boldsymbol{\delta}, \boldsymbol{R}_i, \boldsymbol{t}_i). \tag{26}$$

Step2: Solve projection matrix

```
Undistort control points x_u = x_p − d_p using the estimated parameters A, R, t, δ.
Estimate projection matrix P by minimizing
```

$$\sum_i \sum_j \tilde{\boldsymbol{x}}_{u,i,j} - \boldsymbol{P}_i \tilde{\boldsymbol{X}}_j. \tag{27}$$

Output: Estimated parameters $\boldsymbol{P}$ and the undistorted images.

the following expression:

$$\tilde{\boldsymbol{x}}_p = \boldsymbol{A}\left[\tilde{\boldsymbol{x}}_n + \tilde{\boldsymbol{d}}\right] \tag{19}$$

$$\rightarrow \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \theta & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n + d_x \\ y_n + d_y \\ 1 \end{bmatrix} \tag{20}$$

$$= \begin{bmatrix} f_x(x_n + d_x) + \theta(y_n + d_y) + o_x \\ f_y(y_n + d_y) + o_y \\ 1 \end{bmatrix} \tag{21}$$

$$= \begin{bmatrix} f_x x_n + \theta y_n + o_x \\ f_y y_n + o_y \\ 1 \end{bmatrix} + \begin{bmatrix} f_x d_x + \theta d_y \\ f_y d_y \\ 1 \end{bmatrix} \tag{22}$$

$$= \boldsymbol{A}\tilde{\boldsymbol{x}}_n + \tilde{\boldsymbol{d}}_p \tag{23}$$

$$= \tilde{\boldsymbol{x}}_u + \tilde{\boldsymbol{d}}_p \tag{24}$$

Since lens distortion vector $\boldsymbol{d}$ is computed on the canonical view, considering lens distortion implicitly assumes that we decompose a projection matrix into intrinsic and extrinsic parameters.

To solve a projection matrix considering lens distortion, we first undistort $\boldsymbol{x}$ and then solve the parameters. With correct projection matrix $\boldsymbol{P}$, the following equation should be held for any corresponding points:

$$\boldsymbol{P}\tilde{\boldsymbol{X}} = \tilde{\boldsymbol{x}}_u \tag{25}$$

Since the equation is linear w.r.t. $\boldsymbol{P}$, we can solve $\boldsymbol{P}$ by direct linear transformation. Combining this strategy into typical camera calibration method Zhang [1998, 2000], we derive the following algorithm.

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