Singular and Non-singular Matrix

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Non-Singular

$\mathbf{A}$ is non-singular means that $\mathbf{A}$ is invertible ($\mathbf{A}^{-1}$ exists).

- Can solve $\mathbf{Ax} = \mathbf{b}$ as $\hat{\mathbf{x}} = \mathbf{A}^{-1}\mathbf{b}$.
- The above solution is unique.
- For homogeneous system $\mathbf{Ax} = \mathbf{0}$, the only solution is $\mathbf{x} = \mathbf{0}$.

Singular

$\mathbf{A}$ is singular means that $\mathbf{A}$ is not invertible ($\mathbf{A}^{-1}$ doet not exist).

- Either
  - a solution to $\mathbf{Ax} = \mathbf{b}$ does not exist,
  - there is more than one solution (not unique).
- The homogeneous system $\mathbf{Ax} = \mathbf{0}$ has more than one solution.
  - Infinitely many non-trivial solutions.

Comparison

	Non-singular	Singular
$\mathbf{A}$ is	invertible	not invertible
Columns	independent	dependent
Rows	independent	dependent
$\det(\mathbf{A})$	$\neq 0$	$= 0$
$\mathbf{Ax} = \mathbf{0}$	one solution $\mathbf{x} = \mathbf{0}$	infinitely many solution
$\mathbf{Ax} = \mathbf{b}$	one solution	no solution or infinitely many
$\mathbf{A}$ has	n (nonzero) pivots	$r < n$ pivots
$\mathbf{A}$ has	full rank $r = n$	rank $r < n$
Column space	is all of $\mathbb{R}^n$	has dimension $r < n$
Row space	is all of $\mathbb{R}^n$	has dimension $r < n$
Eigenvalue	All eigenvalues are non-zero	Zero is an eigenvalue of $\mathbf{A}$
$\mathbf{A}^\mathsf{T}\mathbf{A}$	is symmetric positive definite	is only semidefinite
Singular value of $\mathbf{A}$	has n (positive) singular values	has $r < n$ singular values